Variation of latitude, the range of which is apparently between 0″·2 and 0″·7, would not go far towards explaining a discrepancy of 2″·3.

On the Optical Distortion of a Doublet Lens. By Captain E. H. Hills, R.E.

In the May number of *Monthly Notices*, Professor Turner has discussed the distortion of a doublet lens, and has shown that it is at all events probable that it is a negligible quantity over a field of 11° square.

In my investigation on the determination of longitudes by photography (*Memoirs R.A.S.* vol. liii.), it was necessary to satisfy myself, before applying the formulæ of reduction to the star places, that any possible optical distortion of the lens employed did not introduce a measurable factor into the results, and I accordingly carried out some experiments on this point.

The results I arrived at were practically identical with those reached by Professor Turner, but as I approached the subject by a different road it seems desirable to give a short account of the work. The method used was to set up the camera in a fixed position, and, leaving the lens open for a considerable time, a number of star trails were drawn across the plate. The curvature of these trails was then measured and compared with the calculated amount.

For convenience of computation, the camera was so arranged that the projection of the equator fell near the centre of the plate. This is not absolutely necessary, as the rigorous formulæ present no special difficulty; but, as will be seen immediately, the employment of a simple, approximate formula for the curvature does not introduce any appreciable error until the centre of the plate is moved many degrees from the equator.

The theoretical curvature of a star trail on the plate may be derived as follows:

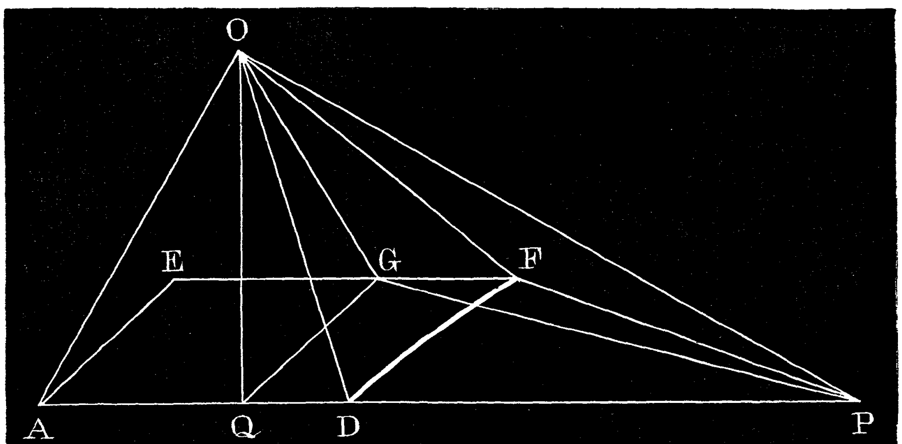

FIG. 1.

Q is the 'centre' of the plate—*i.e.* the foot of the normal from the centre of the lens on the plate.

O is the centre of projection—*i.e.* the optical centre of the lens.

AE is the projection of the equator.

P is the projection of the pole.

DF is any star trail.

The line EF is drawn on the plate parallel to AP at any arbitrary angular distance from it, and QG is parallel to AE.

Let

$$QOG = \theta.$$

$$OQ = \text{radius} = 1.$$

$$QOA = \text{declination of centre of plate} = \phi.$$

$$DOA = \text{declination of star} = \delta.$$

Then GF—QD is the curvature required.

We have

$$QG = \tan\theta.$$

$$QP = \cot\phi.$$

$$\tan QPG = \tan\theta\tan\phi.$$

$$= \tan\alpha.$$

In the triangle GOP

$$OG = \sec\theta.$$

$$OP = \operatorname{cosec}\phi.$$

$$GP = \tan\theta\operatorname{cosec}\alpha.$$

$$\cos GOP = \frac{\sec^2\theta + \operatorname{cosec}^2\phi - \tan^2\theta\operatorname{cosec}^2\alpha}{2\sec\theta\operatorname{cosec}\phi}$$

but

$$\operatorname{cosec}^2\alpha = 1 + \cot^2\alpha = 1 + \cot^2\theta\cot^2\phi.$$

Therefore,

$$\cos GOP = \frac{\sec^2\theta + \operatorname{cosec}^2\phi - \tan^2\theta - \cot^2\phi}{2\sec\theta\operatorname{cosec}\phi}$$

$$= \cos\theta\sin\phi.$$

$$= \cos\beta.$$

In the solid figure OGFP, taking the angles at G, we have

$$OGF = 90°$$

$$FGP = \alpha$$

and angle between planes OGF, FGP $= 90° - \theta$.

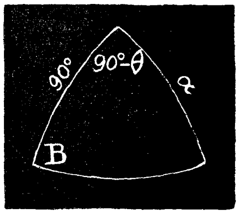

FIG. 2.

Then, if B = angle between planes OGF, OGP,

$$\tan B = \tan \alpha \cos \theta$$
$$= \sin \theta \tan \phi.$$

Again taking the angles at O, we have

$$FOP = 90^\circ - \delta.$$
$$GOP = \beta.$$

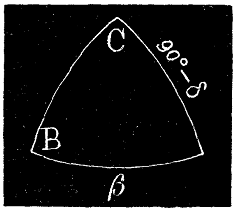

FIG. 3.

Then if C = angle between planes FOP, GOF

$$\sin C = \frac{\sin B \sin \beta}{\cos \delta}$$

and

$$\tan \frac{GOF}{2} = \frac{\sin \frac{B+C}{2}}{\sin \frac{B-C}{2}} \tan \frac{90^\circ - \delta - \beta}{2}$$

$$= \tan \frac{\gamma}{2}.$$

We have therefore finally as the expression for the curvature—

$$GF - QD = \sec \theta \tan \gamma - \tan (\delta - \phi)$$

where

$$(1) \quad \tan \frac{\gamma}{2} = \frac{\sin \frac{B+C}{2}}{\sin \frac{B-C}{2}} \tan \frac{90^\circ - \delta - \phi}{2}.$$

$$(2) \quad \sin C = \frac{\sin B \sin \beta}{\cos \delta}.$$

$$(3) \quad \tan B = \sin \theta \tan \phi.$$

$$(4) \quad \cos \beta = \cos \theta \sin \phi.$$

The angle θ can be selected any convenient magnitude, say 5°, and it is then easy to construct a table of double entry, giving the value of the curvature at any distance from the centre. In practice, however, this is unnecessary. It is obvious that when ϕ is small the expression for the curvature becomes

$$\tan \delta (\sec \theta - 1),$$

and the difference between this approximate value and the true one will remain quite negligible even when ϕ amounts to many degrees.

Thus taking $\theta = 5°$ and values of δ between 0° and 10°, it will be found that, even when $\phi = 8°$, the difference between $\sec\theta\tan\gamma - \tan(\delta-\phi)$, and $\tan\delta(\sec\theta - 1)$ does not amount to a unit in the 7th place of decimals, *i.e.* does not exceed $0''\cdot02$.

In taking the test plate there can be no possible difficulty in so adjusting the camera that the centre lies near the equator, and the theoretical curvature of a trail can then be deduced in the above simple manner.

As an example of the method the results obtained with an actual plate may now be given.

The lens in this case was a Zeiss anastigmat of 377 mm. focal length. The micrometer used read to ·001 mm., equivalent to $0''\cdot6$ of arc on the plate. An error of four times this amount, or between $2''$ and $3''$, is quite possible in any individual measurement.

The measures may therefore be taken as qualitative rather than quantitative, as it is obvious on inspection of the residuals that there is no measurable distortion over the field taken—namely, to a distance of about 9° from the centre, equivalent to a square plate of about $12\frac{1}{2}°$.

Table Giving Results of Test of Zeiss Anastigmatic Lens.

$$\begin{cases} r = 377 \text{ mm.} \\ \theta = 5° \\ \phi = 24' \end{cases}$$

Star.	δ	Distance from Centre of Plate.	Curvature in Millimetres. Observed.	Calculated.	O−C	O−C in Arc.
	° ′	° ′				″
1	−1 16	0 52	·030	·030	± ·000	± 0·0
2	−2 0	1 36	·049	·050	− ·001	−0·6
3	−2 40	2 16	·067	·066	+ ·001	+0·6
4	+3 0	3 24	·080	·075	+ ·005	+2·9
5	+4 3	4 27	·099	·102	− ·003	−1·7
6	−4 54	4 30	·121	·124	− ·003	−1·7
7	+5 52	6 16	·149	·148	+ ·001	+0·6
8	−5 58	5 34	·151	·150	+ ·001	+0·6
9	+6 15	6 39	·165	·159	+ ·006	+3·4
10	+7 23	7 47	·188	·186	+ ·002	+1·1

As Prof. Turner has pointed out, the effect of refraction is practically eliminated by taking the measurements in the form of differences of curvature of two trails. This was done in the above example. The trail of a known star (δ *Orionis*), near the centre of the plate, was taken as the fiduciary line from which the measurements were made, and the deduced curvatures were

then corrected by adding or subtracting the calculated curvature of this trail. These residuals will no doubt at first sight appear large to those accustomed to work with instruments of long focal length, but it must be borne in mind that these same quantities expressed in linear measurement, which is really fairer for purposes of comparison, are extremely small. The unit of measurement used—namely, ·001 mm.—certainly represents the extreme limit of accuracy, if indeed it be not beyond it, that can be obtained from any stellar photograph. If a more liberal standard of accuracy were adopted the appearance of the residuals would be correspondingly improved. Thus taking the standard of the astrographic plates and carrying the measurements, as is done with them, only to a limit of ·005 mm., it will be seen at once that nearly all the residuals would be zero.

It may further be noted that, as any distortion which is linear in r will disappear in the reduction to rectilinear co-ordinates, we are entitled to assume that value of r which will make the sum of the residuals a minimum, provided always that there be sufficient stars on the plate to give a real value of r. In this example, as all the residuals are practically within the limits of possible errors of measurement, a correction of the assumed value of r would be unnecessary and hardly justifiable.

List No. 12 *of Nebulæ discovered at Lowe Observatory, Echo Mountain, California, for* 1900·0. By Lewis Swift.

No.	Date 1898.	R.A. h m s	Dec. ° ′	Description.
1	May 24	0 4 10	−32 49·9	vF, vS, R, unequal D ✱ n.
2	24	0 5 30	− 7 58·3	eeeF, vS, R, bet $7\frac{1}{2}^m$ ✱ n and 9^m ✱ s, eee dif.
3	22	0 29 40	−30 32·9	eeeF, S, R, wide D ✱ close p point to it. Not 174.
4	Nov. 19	0 53 0	−17 ?	pF, vS, $7\frac{1}{2}^m$ ✱ np, F ✱ near s p.
5	1897. May 24	1 0 0	−27 56·4	eF, pS, close to 3 st like belt of Orion.
6	Oct. 12	1 4 0	−29 6·6	cB, pS, R, 3 8^m st near.
7	Sept. 20	1 28 0	−14 0·8	eeF, R, S, lE, 8^m ✱ n, e dif.
8	Oct. 10	1 54 15	−28 16·5	eeF, S, R, 8^m ✱ S, lf.
9	Previous	11 49 ?	− 5 ?	eeF, lE, v small, 3 B st in line n, also circle of st n. Saw it twice, failed once.
10	June 24	12 15 5	+61 15·0	eeeF, S, $7\frac{1}{2}$ and 5^m st in field, p of 2. One of my faintest nebulæ.
11	24	12 15 35	+61 15·0	vF, pL, R, $7\frac{1}{2}^m$ ✱ south, f of 2.
12	Aug. 19	15 29 ?	+ 6 21·0	eeeF, L, R, eee dif.
13	19	15 50 ?	+ 6 19·0	eF, S, R, bet 8^m ✱ f, and curve of st p.
14	16	19 41 20	−33 34·0	eeeF, eeS, eee dif sev F st near.